Demand bidding construction for a large consumer through a hybrid IGDT-probability methodology

Kazem Zare a, Mohsen Parsa Moghaddam b,*, Mohammad Kazem Sheikh El Eslami b

a Tabriz University, Tabriz, P.O. Box: 51666-15813, Iran
b Tarbiat Modares University, Tehera, Iran

1. Introduction

1.1. Aim

In an electricity market, the consumers procure their electricity needs from the market or from self-production facilities. In this paper, we consider a large consumer that participates in the day-ahead and adjustment markets to procure its electricity demand while controlling the risk of cost volatility [1] in the presence of market price uncertainty. We also consider that this consumer may procure part of its electricity demand using a self-production facility.

By using the forecasted prices of electricity in these two markets, the total procurement cost can be computed as an expected value. Since the hourly market prices are normally distributed, the total procurement cost is uncertain for the consumer and can be presented by a normal distribution function. It is considered that the hourly prices in both markets have a normal probability distribution and these prices are correlated with each other. Thus, we propose a method to find a robust bidding curve against experiencing the procurement cost higher than the expected one. The proposed method allows constructing a robust bidding strategy against the risk associated with uncertain day-ahead and adjustment market prices. Therefore, considering a short-term planning horizon of 24 h, we focus on the problem of deriving the optimal hourly day-ahead bidding curves that the consumer should submit to day-ahead market.

1.2. Procedure

Uncertainties are generally quantified by mathematical abstractions such as probability density functions or fuzzy logic membership. Information Gap Decision Theory (IGDT) is an alternative approach for decision making under uncertainty that makes minor assumptions on the structure of the uncertainty. IGDT models assume neither probabilistic density nor fuzzy membership functions, and focus on the disparity between what is actually known and what could be known. A brief description of the IGDT technique can be found in the Appendix.

Decisions for procurement of electricity are related to the market price volatilities. By considering a joint normal distribution function for 24 h of day-ahead and adjustment markets, the final procurement cost will have a normal probability distribution function [2]. Here, we formulate the uncertainty of the expected procurement cost based on the concepts derived from IGDT [3], and then the demand bidding curve is constructed by using First Order Reliability Method (FORM) [4]. In this method, the strategy relies on maximizing the robustness of the bidding strategy against high procurement cost.
Conceptually, the IGDT-based model proposed in this paper is similar to other models that explicitly consider risk, such as mean-variance models and scenario-based models that include risk control through some risk metrics. The IGDT-based modeling does not require any assumption of the nature of uncertainty while, for example the mean-variance models need some assumptions on the uncertain random variables and the scenario-based models explicitly require a procedure to generate scenarios based on some uncertainty assumptions.

### 1.3. Literature review

Bidding strategy construction of generation side with their operation and management strategies has been dealt in several papers and some different methods have been addressed [5–8], but there are few works addressing the bidding curve construction for consumers.

The possible scenarios pertaining to the implementation of demand bidding in electricity markets are evaluated in [9]. The demand bidding problem has been addressed in [10], where the demand is considered uncertain and the price known, using a stochastic dynamic programming method for optimal purchase allocation in multi-markets. The optimal purchase allocation and demand bidding are also discussed in [11], where the price volatility is explicitly considered. Analytical solutions for the optimal allocation considering linearly correlated and independent prices are derived based on conditional stochastic programming in [11]. Obtaining bidding curves for a price-taking retailer is addressed in [13] based on game theory and market distribution functions.

### 1.4. Contributions

This paper applies a novel method based on the concepts of probability distribution theory, IGDT, and FORM to derive the bidding strategy for a large consumer. The proposed method enables the consumer to build a robust and risk-averse bidding curve against high procurement cost and relatively high pool prices.

### 1.5. Paper organization

The remaining of the paper is organized as follows: Section 2 is devoted to analyze uncertainty issues and formulate the demand bidding problem faced by a large consumer using IGDT and finally construct the demand bidding curve using FORM. Numerical studies are provided in Section 3 to show the interest of the proposed method. Section 4 concludes the paper. Finally, an appendix provides background on IGDT.
2. Formulation

2.1. Price modeling

Consider that the hourly prices of the day-ahead and the adjustment markets be denoted respectively by:

\[ \lambda^D = \left[ \lambda_1^D, \ldots, \lambda_T^D \right], \lambda^A = \left[ \lambda_1^A, \ldots, \lambda_T^A \right], \tag{1} \]

which can be conveniently arranged as:

\[ \lambda = \left[ \lambda_1^D, \ldots, \lambda_T^D, \lambda_1^A, \ldots, \lambda_T^A \right]^T \tag{2} \]

Since the hourly prices in day-ahead and adjustment markets are assumed to have a joint normal probability distribution function with average \( \bar{\lambda}_i \) and standard deviation \( \sigma^2_i \), so we can define the prices as following:

\[ \lambda_i \sim N(\bar{\lambda}_i, \sigma^2_i), \quad i = 1, \ldots, 2T. \tag{3} \]

2.2. Procurement cost

The procurement cost incurred by the consumer is composed of the cost of electricity procurement from day-ahead and adjustment markets and the operation cost of self-production facility, which is computed as:

\[ C(Q, \lambda) = Q^T \lambda + \sum_{t=1}^{T} \sum_{h=1}^{N} c^S_{h,t} E^S_{h,t} \tag{4} \]

where, \( Q = [q_1^D, \ldots, q_T^D, q_1^A, \ldots, q_T^A]^T \), denotes the procured energy from day-ahead and adjustment markets.

The operation cost of the self-production facility is represented by a three-block piecewise linear function [14] as depicted in Fig. 1.

The total energy supplied by self-production facility is computed as,

\[ q^S_t = \sum_{h=1}^{N} E^S_{h,t}, \quad t = 1, \ldots, T \tag{5} \]

where

\[ Q^S = \left[ q^S_1, \ldots, q^S_T \right] \tag{6} \]

represents the vector of procured energy from self-production facility.

From probability point of view, since the price variables have normal probability distribution, the energy procurement cost function will related to a jointly normal probability distribution, which can be identified with the following parameters as:

\[ C \sim N(\bar{C}, \sigma^2_C) \tag{7} \]

where,

\[ \bar{C} = Q^T \lambda + \sum_{t=1}^{T} \sum_{h=1}^{N} c^S_{h,t} E^S_{h,t} \tag{8} \]

\[ \sigma^2_C = \sum_{i=1}^{2T} \sum_{j=1}^{2T} q_i q_j V(i,j) \tag{9} \]

where, \( \bar{C}, \sigma^2_C \) and \( V \) denote the average or expected procurement cost, variance of procurement cost and the variance–covariance matrix between hourly prices in both day-ahead and adjustment markets, respectively. It should be noted that \( \bar{C} \) is the result of minimizing the procurement cost function using the forecasted pool prices \( \bar{\lambda} \).

2.3. Uncertainty model

Here, we consider that because of the uncertainty in hourly prices of day-ahead and adjustment markets, the expected procurement cost has uncertainty. We express this uncertainty by the following model:

\[ U(\alpha, C^{exp}) = \left\{ C^{exp} : |C^{exp} - \bar{C}| \leq \alpha \sigma_C \right\}, \quad \alpha \geq 0, \tag{10} \]

where \( C^{exp} \) denotes the expected procurement cost.

Fig. 2 shows the typical forecasted cumulative distribution of the procurement cost with the uncertainty in its average value.

Without considering the uncertainty, the expected value of the cost is equal to the average cost, \( \bar{C} \), which can be defined as following:

\[ E(c) = \frac{1}{2\pi \sigma_C} \int_{-\infty}^{+\infty} c \exp \left( -\frac{(c - \bar{C})^2}{2\sigma_C^2} \right) dc = \bar{C} \tag{11} \]

But by considering the uncertainty on average cost, the result of (11) will not be equal to \( \bar{C} \).
2.4.Robustness problem

In order to have a bidding strategy by representation of the cost uncertainty based on IGDT, the consumer might be interested in determining the maximum value of the expected cost using (10). Note that this situation represents the worst cost for the consumer. Then, we can express it as:

\[
\text{maximum expected cost} = \max(C_{\text{esp}}) = \overline{c} + \sigma_c
\]  

Substituting \(C\) and \(\sigma_c\) from (8) and (9) in (12) yields that:

\[
\text{maxE} (c) = Q^t \lambda + \sum_{t=1}^{T} \sum_{h=1}^{N} C_h^t E_h^t + \alpha \sqrt{\sum_{i=1}^{2T} \sum_{j=1}^{2T} q_i q_j V(i,j)}
\]  

Using (13), \(\alpha\) can be computed for a given procurement cost target \(C_t\) as

\[
\alpha(C_t) = \frac{C_t - \left( Q^t \lambda + \sum_{t=1}^{T} \sum_{h=1}^{N} C_h^t E_h^t \right)}{\sqrt{\sum_{i=1}^{2T} \sum_{j=1}^{2T} q_i q_j V(i,j)}}
\]  

Then, based on IGDT we can derive the robustness function and consequently a robust demand bidding strategy by providing the largest possible \(\alpha\) for a given cost target \(C_t\) as following optimization problem,

\[
\hat{\alpha}(C_t) = \max_{Q_0^t} \alpha(C_t) = \max_{Q_0^t} \frac{C_t - \left( Q^t \lambda + \sum_{t=1}^{T} \sum_{h=1}^{N} C_h^t E_h^t \right)}{\sqrt{\sum_{i=1}^{2T} \sum_{j=1}^{2T} q_i q_j V(i,j)}}
\]  

subject to:

\[
q_t + q_{t+T} + q_t^s = D_t , \quad t = 1, \ldots, T
\]  

\[
q_t^s = \sum_{h=1}^{N} E_h^t , \quad t = 1, \ldots, T
\]  

\[
0 \leq E_h^t \leq (E^{\text{MAX}}_h - E^{\text{MIN}}_h)V_{h-1} , \quad t = 1, \ldots, T , \quad h = 2, \ldots, N
\]  

\[
0 \leq E_h^t \leq E^{\text{MAX}}_h V_t , \quad t = 1, \ldots, T
\]  

where, \(D_t\) is the demand to be served at hour \(t\). Expression (15) represents the robustness function to be maximized is constrained by (16) to (19). Equations (16) express the power balance between the load and the procured electricity from day-ahead and adjustment markets and self-production facility. Constraints (17) to (19) are related to the operation cost model of the self-production facility that is represented by a three-block piecewise linear function. The binary variables in (18) and (19) express that the block \(h\) of the operation model of self-production facility will be selected when \(p_{h-1,t}^c = p_{h-1,t}^{\text{MAX}}\), and \(p_{h,t}^c\) will be zero if \(p_{h-1,t}^c < p_{h-1,t}^{\text{MAX}}\).

The results of the solution of (15)–(19) for a given cost target \(C_t\) is

\[
Q^*, Q^s, \hat{\alpha}(C_t)
\]  

where subindex \(l\) denotes cost target \(l\).

2.5. Construction of the bidding curve

The typical bidding curve for day-ahead market is shown in Fig. 3. It is obvious that to derive this bidding curve, we need to determine the quantity–price pairs \((q, \hat{\lambda})\). So, we should find the best closest price pairs related to a given cost target \(C_t\) and its related optimal strategy \(Q^*, Q^s\).

For the defined decision variables \(Q^*, Q^s\), the cost function involves a number of random variables \(\hat{\lambda}\). The random variables involved belong to 2T-dimensional space and can be calculated using the joint probability density function of all random variables involved by means of the following integral:

\[
C_t = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} C(\lambda_1, \ldots, \lambda_{2T})f(c)d\lambda_1 d\lambda_2 \cdots d\lambda_{2T}
\]  

where \(f(c)\) denotes the joint probability distribution function of the procurement cost function.

Note that (21) represents the definition of the expected value of the procurement cost and we should find the related variables \(\hat{\lambda}\) while the integral in (21) be equal to a given cost target \(C_t\). Direct integration of (21) is difficult because, (a) it requires the definition of the joint probability of all variables involved, and (b) the evaluation of a 2T-dimensional integral over a non-linear probability is generally complex [15].

Rather than using approximate (and numerical) methods to perform the integration required in (21), the non-normal dependent distributions can be transformed into equivalent dependent normal distributions that can be integrated. We apply the First Order Reliability Method, which is widely used in engineering designs [16,17], to find these price pairs. Thus, using the appropriate transformations, price pairs can be found by solving the following optimization problem:

\[
\hat{\beta} = \min_{\lambda} \sqrt{\sum_{i=1}^{2T} \hat{\lambda}_i^2}
\]  

s.t.

\[
\sum_{t=1}^{T} \sum_{h=1}^{N} C_h^t E_h^t + Q^* \lambda = C_t
\]  

\[
Z = L^{-1}(\hat{\lambda} - \lambda)
\]  

Fig. 3. The typical bidding curve in day-ahead market.
\[ LL^T = V \]  
\[ Z^T = [z_1, \ldots, z_T]; \]  
where, \( V \) is the variance–covariance matrix between the prices of two markets, and matrix \( L \) is the Cholsky transformation of \( V \). The variables \( Z \) are random variables with independent normal distributions. Equation (22) provides an optimization problem that the difference between the unknown price pairs and the forecasted ones should be minimized subject to the predefined procurement strategy of a given cost target \( C_i \) in transformed space. Equation (23) represents the procurement cost function with its related and predetermined procurement strategy for a given cost target \( C_i \). The transformation methodology is provided by (24) that converts \( \lambda \) into \( Z \) using the cholsky transformation [18]. This technique represented in (24) [19].

The minimization objective function of (22) is called the safety or reliability index, whereas the optimal vectors \( \lambda^* \) and \( z^* \) are the design points or points of maximum likelihood in the initial and transformed space, respectively, which are related by (24). The solution of problem (22) provides the closest price pairs \( l^* \) for the procurement strategy \( (Q^*, Q^S^*) \) of any given cost target \( C_i \) to construct the bidding curve.

### 2.6. Model results

Finally by using the results of both optimization problems in (15) and (22) with their related constraints, the bidding curves will be found as following:

\[
\lambda_i^D (q_{i}^{D^*}) \quad \forall i, \quad \text{for} \quad l = 1, \ldots, L
\]

### 2.7. Computational size

The robustness problem addressed by (15) and the problem addressed in (22) are non-linear programming problems that can be solved using CONOPT under GAMS software [20]. Table 1 provides the size of these problems, which is expressed as the number of real variables and constraints.

### 3. Case study

In this section, numerical simulations are reported to illustrate the working of the proposed method. The considered planning horizon is one day divided into 24 h.

#### 3.1. Data

The load profile of the consumer and the forecasted prices estimates for the day-ahead and the adjustment markets are depicted in Figs. 4 and 5, respectively. The adjustment market prices are greater than day-ahead prices in most hours, which is typical in the Iberian electricity market [21].

The operation cost of the self-production facility is modeled by three piecewise linear approximated model. The data of the self-production facility are provided in Table 2. The variance–covariance matrices for both the day-ahead and the adjustment markets are provided in Tables 3 and 4, respectively.

### Table 1

<table>
<thead>
<tr>
<th>Problem (15)</th>
<th>Problem (22)</th>
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<tr>
<td># of real variables</td>
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<td># of constraints</td>
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### Table 2

Data for the self-production facility.

| Capacity | 60 | MW |
| Minimum power output | 0 | MW |
| \( c_1 \) | 80 | $/MWh |
| \( c_2 \) | 90 | $/MWh |
| \( c_3 \) | 105 | $/MWh |
| \( E_{MAX}^l \) | 25 | MW |
| \( E_{MAX}^s \) | 45 | MW |
| \( E_{MAX}^c \) | 60 | MW |

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2. For 180 cost target with a fixed step, which are higher than the minimum cost, the problem (15)–(19) is simulated which results represent the robustness level and procurement percentage from day-ahead market. In this simulation, the cost step for $C_t$ is considered to be equal to $3000.

3. Using the results of the previous optimization, the problem (22)–(26) is solved to find the related price pairs to construct the bidding curves (27) for the day-ahead market.

### 3.2. Results

The minimum expected procurement cost by considering the forecasted prices, is equal to $371,939. This value is the result of minimizing (4) with the best estimation of the markets prices. From the uncertainty point of view, we can say that $\tilde{a}(371,939) = 0$.

The next step is to solve problem (15)–(19) for different values of the cost targets $C_t$. The solution results provide a robustness value $\tilde{a}$ and an optimal vector of hourly power quantities to be procured from the day-ahead market. Bidding curves are constructed computing $\lambda$ using (22)–(26) once procurement strategies are obtained from solving (15)–(19).

The robustness value $\tilde{a}$ versus the cost target, $C_t$, is depicted in Fig. 6. It is shown that the robustness increases as the cost target $C_t$ grows.

The resulting bidding curves for three selected hours are shown in Figs. 7–9. Note that in these figures the vertical axis indicates the percentage of the hourly demand that is procured from the day-ahead market and the horizontal axis indicates the computed day-ahead market prices. As expected, an increase in price implies a smaller portion of demand obtained from the day-ahead market. Note that similar behavior is observed for the rest of hours.

### Table 4

#### Variance–covariance matrix for the day-ahead market.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 1 | 22 | 17 | 13 | 11 | 11 | 14 | 13 | 12 | 14 | 12 | 12 | 11 | 10 | 10 | 9 | 6 | 4 | 1 | -2 | -5 | -4 | 8 | 11 | 7 | -2 | 1 |
| 2 | 21 | 17 | 16 | 15 | 14 | 11 | 13 | 10 | 12 | 10 | 9 | 7 | 9 | 4 | 2 | 0 | -3 | -2 | -8 | 11 | 3 | 11 | -1 | 1 |
| 3 | 18 | 16 | 18 | 17 | 16 | 11 | 13 | 11 | 13 | 12 | 11 | 9 | 5 | 4 | 2 | 0 | -3 | -2 | -8 | 11 | 3 | 11 | -1 | 2 |
| 4 | 15 | 17 | 21 | 22 | 19 | 12 | 18 | 13 | 12 | 11 | 11 | 10 | 12 | 5 | 4 | 1 | -1 | -2 | 1 | 0 | 4 | 0 | -2 | 1 |
| 5 | 16 | 15 | 22 | 26 | 21 | 13 | 12 | 13 | 12 | 11 | 12 | 11 | 7 | 5 | 2 | 0 | -1 | -4 | -2 | -2 | 0 | 0 | -1 | 1 |
| 6 | 14 | 14 | 16 | 19 | 21 | 20 | 12 | 13 | 11 | 11 | 11 | 11 | 9 | 6 | 4 | 2 | -1 | -1 | 2 | 0 | 3 | -2 | 0 | 2 |
| 7 | 11 | 11 | 11 | 13 | 12 | 13 | 12 | 10 | 11 | 9 | 9 | 7 | 5 | 4 | 3 | 2 | 0 | 0 | 4 | 1 | 1 | -3 | -1 |
| 8 | 13 | 13 | 13 | 13 | 13 | 13 | 23 | 23 | 21 | 23 | 20 | 19 | 15 | 11 | 7 | 6 | 4 | 3 | 4 | 15 | 14 | 9 | -1 | 1 |
| 9 | 11 | 10 | 11 | 12 | 11 | 10 | 21 | 26 | 27 | 25 | 23 | 18 | 14 | 13 | 8 | 7 | 5 | 9 | 4 | 6 | 20 | 18 | 8 | -1 | 2 |
| 10 | 14 | 12 | 13 | 12 | 11 | 12 | 21 | 26 | 27 | 25 | 23 | 18 | 14 | 13 | 8 | 7 | 5 | 9 | 4 | 6 | 20 | 18 | 8 | -1 | 2 |
| 11 | 13 | 10 | 12 | 11 | 12 | 11 | 9 | 20 | 25 | 34 | 35 | 32 | 27 | 22 | 14 | 12 | 9 | 7 | 11 | 33 | 32 | 12 | 1 | 6 |
| 12 | 12 | 9 | 11 | 12 | 11 | 9 | 19 | 23 | 30 | 32 | 32 | 28 | 23 | 16 | 14 | 10 | 8 | 11 | 29 | 28 | 8 | 2 | 6 |
| 13 | 9 | 7 | 9 | 10 | 11 | 9 | 7 | 15 | 18 | 25 | 27 | 28 | 27 | 23 | 16 | 15 | 11 | 9 | 10 | 23 | 22 | 5 | 2 | 5 |
| 14 | 6 | 4 | 5 | 6 | 7 | 5 | 11 | 14 | 20 | 22 | 23 | 23 | 22 | 16 | 14 | 11 | 10 | 11 | 23 | 25 | 5 | 3 | 7 |
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| 17 | -2 | -2 | 0 | 1 | 2 | 2 | 2 | 4 | 5 | 7 | 9 | 10 | 11 | 11 | 12 | 14 | 13 | 13 | 11 | 9 | 2 | 5 | 5 |
| 18 | -5 | -4 | -3 | -1 | 0 | -1 | 1 | 1 | 3 | 4 | 6 | 7 | 8 | 9 | 10 | 11 | 13 | 15 | 14 | 11 | 3 | 6 | 5 |
| 19 | -5 | -4 | -2 | -1 | 1 | 0 | 4 | 6 | 11 | 11 | 12 | 12 | 10 | 11 | 11 | 12 | 13 | 15 | 19 | 27 | 26 | 10 | 9 | 4 |
| 20 | 8 | 8 | 8 | 0 | 1 | 2 | 0 | 4 | 15 | 20 | 35 | 33 | 32 | 29 | 23 | 21 | 17 | 14 | 11 | 14 | 27 | 80 | 93 | 42 | 19 | 26 |
| 21 | 11 | 13 | 11 | 0 | -4 | 0 | 1 | 14 | 18 | 36 | 32 | 28 | 22 | 25 | 16 | 13 | 9 | 11 | 26 | 93 | 127 | 65 | 26 | 34 |
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| 24 | 1 | 1 | 2 | 0 | 0 | -1 | 1 | 2 | 6 | 6 | 6 | 5 | 7 | 6 | 5 | 5 | 9 | 26 | 34 | 17 | 13 | 18 |

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Table 5
Contemporaneous variance–covariance matrix for day-ahead and adjustment markets.

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Fig. 6. Robustness versus cost targets.

Fig. 7. Day-ahead bidding curve for hour 6.

Fig. 8. Day-ahead bidding curve for hour 17.

Fig. 9. Day-ahead bidding curve for hour 22.

Please cite this article in press as: Zare K, et al., Demand bidding construction for a large consumer through a hybrid IGDT-probability methodology, Energy (2010), doi:10.1016/j.energy.2010.03.036
Finally, Fig. 10 shows the calculated day-ahead prices for three cost targets from solving (22)–(26). This figure shows that the hourly price deviations are not linear and are correlated with other hours based on variance–covariance matrix.

4. Conclusion

A novel hybrid decision-making method based on IGDT and probability concepts is presented to construct the robust bidding strategy for a large consumer in day-ahead market. The uncertainty of the procurement cost is modeled using the variance–covariance matrix of the prices of day-ahead and adjustment markets. The proposed method results in a robust bidding strategy against procurement costs higher than the expected one. This paper shows through a realistic case study the interest of the proposed methodology to derive robust bidding strategies for a large consumer.

Acknowledgment

The authors would like to thank Prof. Antonio J. Conejo and Dr. Roberto Minguez from Castilla-La Mancha University in Ciudad Real, Spain, for their valuable comments and relevant observations.

Appendix. IGDT background [3]

IGDT is based on quantitative models and provides numerical decision-support assessment. However, this theory is not a closed computational methodology. Rather, the quantitative assessments assist the decision maker to evaluate options, to develop strategies, and to evolve preferences in light of the analysis of uncertainties, expectations and demands. This method focuses on the disparity between what is known and what could be known [3]. In other words, IGDT models the error between the actual and forecasted parameters. In this method, the choice of uncertainty parameters is based on a maximizing robustness or a minimizing opportunity rule. The fact that the uncertainty parameters are initially unspecified, makes IGDT different from other decision-making approaches such as stochastic programming [23].

IGDT can help the decision maker to recognize priorities, evaluate risks and opportunities, and make informed decision. Uncertainties may be pernicious and leads to high cost, or propitious and leads to windfall benefit. IGDT addresses these two conflicting issues using two immunity functions: robustness and opportunity. For a set of decision variables \( Q \) and uncertainty parameter \( a \), we can verbally express the robustness and opportunity functions \( \hat{\alpha} \) and \( \hat{\beta} \) as follows, respectively:

\[
\hat{\alpha} = \max_a (\alpha : \text{minimal requirements are always satisfied}) \tag{28}
\]

\[
\hat{\beta} = \min_a (\alpha : \text{sweeping success is sometimes enabled}) \tag{29}
\]

The robustness function addresses the pernicious face of uncertainty, and expresses the greatest level of uncertainty at which the specified minimal requirements are always satisfied. In this paper, the robustness function is the degree of resistance to uncertainty and immunity against high procurement cost, then, a large value of \( \hat{\alpha} \) is desirable.

The opportunity function addresses the propitious face of uncertainty and evaluates the possibility of benefits. Here, in the case of this problem, \( \hat{\beta} \) is the minimum value of \( a \) which can be tolerated in order to enable the possibility of low procurement cost as a result of decisions \( Q \).

The robustness and opportunity functions are quantitative, but numbers are not enough. The decision maker must make value judgments: how much robustness to the perniciously uncertain parameter is needed, and how much should the opportunities from the uncertainty be facilitated? The answers to these questions can not be unique or algorithmic. They can at best be qualitative and imprecise. But the responsible decision maker must make a connection between quantitative decision analysis and qualitative, linguistic and even subjective values comprising the context of the decision.

The expressions (28) and (29) should be tailored to the specific problem addressed. In the problem addressed in this paper, only the robustness function is utilized.

An IGDT decision problem is specified by three components: system model, performance requirements and uncertainty model.

A. System model

For a set of decision variables \( Q \), uncertainty parameter \( a \), and uncertain parameter \( \lambda \), the system model \( C(Q, \lambda) \) expresses the input/output structure of the system to which the decision is applied. In this paper, the system model is the procurement cost function that a large consumer is faced with.

B. Performance requirements

The performance requirements describe the requirements or anticipations from the system or problem and can be expressed as cost or other functions. These requirements are evaluated based on robustness and opportunity functions are defined according to equations (28) and (29). It is worth mentioned that these functions should be tailored to the specific problem addressed. So, we can express the robustness function for the procurement problem as follows:

\[
\hat{\alpha} = \max_a (\alpha : \text{maximum procurement cost is not higher than a given cost target}) \tag{30}
\]

The robustness function expresses the greatest level of uncertainty at which the procurement cost cannot be greater than a given value \( C_i \). Therefore we can define it mathematically through an optimization problem:
\[ \hat{a}(C_J) = \max_a \{ \alpha : \max_Q (Q, \lambda) \leq C_J \} \]  

(31)

If \( \hat{a}(C_J) \) is large the decision is robust and insensitive to the uncertainties. On the other hand, if \( \hat{a}(C_J) \) is small, the decision is fragile and it does not yield generally consistent decisions.

C. Uncertainty model

The uncertainty can be modeled by an info-gap model. Info-gap model of uncertainty \( U(\alpha, C_{\text{exp}}) \) embodies the prior information about the uncertain parameter \( C_{\text{exp}} \). In this paper, it is considered that the expected value of the total procurement cost has uncertainty and the uncertainty is modeled by the following equation:

\[ U(\alpha, C_{\text{exp}}) = \{ C_{\text{exp}} : |C_{\text{exp}} - C| \leq \alpha \sigma_C, \alpha \geq 0 \}. \]  

(32)

References