Optical filtering properties of inhomogeneous isotropic slab waveguides

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1. Introduction

Nowadays optical integrated circuits have key role for increasing the speed of signal processing in electronic and communication engineering. Usually in this domain the large band-gap material such as photonic crystals or dielectric materials are used. According to similarity between electronic and optical engineering in realization of same engineering purposes, optical layered media has important role as a substrate or device structures [1]. Periodic layered media are a special class of layered media in which layers of dielectric material are stacked in a periodic fashion. The simplest example of a periodic medium consists of alternating layers of two different materials with equal thickness. Wave propagation in these media exhibits many interesting and potentially useful phenomenon. These include Bragg reflectors, holography and optical filtering. The diffraction of X-rays in crystals is a good example of wave propagation in periodic layered media [1]. In addition to the existence of these naturally occurring periodic materials, periodic layered media can also be grown artificially by using various techniques, including molecular beam epitaxy (MBE), metal organic chemical vapor phase deposition (MOCVD) and atomic layer epitaxy (ALE).

The necessary calculation for investigation of these structures can be performed numerically [2,3] and analytically [4–7]. The optical layered media can be divided into two groups and are homogeneous and inhomogeneous media. In homogeneous media the index of refraction is constant versus position in each segment. But in inhomogeneous media the index of refraction distribution is non-uniform versus position. In this paper, we will investigate the filtering properties of inhomogeneous-layered media for three-layer structure. We try to give the exact treatment for reflection and transmission coefficients [8,9].

The organization of this paper is as follows.

In section 2, the optical filtering properties of TM-mode electromagnetic waves travelling in inhomogeneous slab waveguides is investigated. In this section the theoretical background is presented. Result and discussion is presented in section 3. Finally, the paper ends with a conclusion.

2. Inhomogeneous media and optical filtering properties for TM-mode

Fig. 1 is used as optical filter for TM-mode electromagnetic waves.

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where

\[ H_Y(x) = \begin{cases} \frac{Ae^{-ik_0x} + Be^{ik_0x}}{CP(x) + DQ(x)}, & x < -\frac{L}{2} \\ \frac{C}{2} & -\frac{L}{2} < x < \frac{L}{2} \end{cases} \]  

(9)

The functions \( P(x) \) and \( Q(x) \) in Eq. (9) corresponds to the first and the second type of Hermite polynomials and functions respectively and are given as

\[ P(x) = \sqrt{\varepsilon(x)}e^{-x^2/4}H_n \left( x/\sqrt{2} \right), \]

(10)

\[ Q(x) = e^{-x^2/4}Q_n \left( x/\sqrt{2} \right). \]

Using the boundary condition for \( x = \pm L/2 \), we obtain the reflection and transmission coefficients as

\[ r = B/A, \quad t = F/A, \quad \text{where} \]

\[ A = [Q'(-L) - ik_0Q(-L)][P'(L) + ik_0P(L)] - [Q'(L) + ik_0Q(L)][P'(-L) - ik_0P(-L)], \]

\[ B = [P'(L) - ik_0P(L)]Q'(L) + ik_0Q(L) - [Q'(-L) + ik_0Q(-L)][P'(L) + ik_0P(L)], \]

\[ F = [P'(L) + ik_0P(L)][Q'(-L) - ik_0Q(-L)] - [Q'(L) - ik_0Q(L)][P'(-L) - ik_0P(-L)]. \]

Table 1 shows the \( P(x) \) and \( Q(x) \) for \( n = 2,3,4 \). Using Eq. (11), we can obtain the reflection and transmission coefficients of the large set of exactly solvable inhomogeneous media given in Eqs. (2–6,7). Discussion about our proposed structures are given in section 3.

3. Results and discussion

In this section, we will concentrate on four special cases for given \( U(x) = x^2/2 \), which was assumed in section 2.

As a first example, we consider the \( \varepsilon(x) = \varepsilon_0 \).

Case a) \( \varepsilon(x) = \varepsilon_0 \)

Using Eq. (11) and \( \varepsilon(x) = \varepsilon_0 \), we obtain the following function for index of refraction, \( P(x) \) and \( Q(x) \) as

\[ n^2(x) = n_0^2 - x^2/4, \]

(12)

\[ P(x) = e^{-x^2/4}H_n \left( x/\sqrt{2} \right), \]

\[ Q(x) = e^{-x^2/4}Q_n \left( x/\sqrt{2} \right). \]

Our simulated results for this case are shown in Fig. 2. Fig. 2a shows the reflection and transmission coefficients for \( n=2,3,4 \) versus incident light wavelength. As we see, with increasing \( n \) the reflection and transmission coefficients will change very fast and one feel that the degree of the designed filter is increased. The reflection and transmission coefficients versus wavelength and incoming light medium index of refraction are demonstrated in Fig. 2b.
Figure 2 (online color at www.interscience.wiley.com) Case a)
Figure 3 (online color at www.interscience.wiley.com) Case b)
Figure 4 (online color at www.interscience.wiley.com) Case c)
Figure 5 (online color at www.interscience.wiley.com) Case d)
Also, the reflection and transmission versus wavelength, outgoing index of refraction and medium length is shown in Figs. 2c, 2d. Finally, we give the bandwidth of our designed filter as incoming medium, outgoing medium index of refractions and medium length in Figs. (2e, 2f, 2g). As we see from Eq. (5), the bandwidth of our proposed structure strongly depends on length of the inhomogeneous media.

**Case b) $\varepsilon(x) = \varepsilon_0 e^{-\alpha x^2}$**

As a second example, we consider the $\varepsilon(x) = \varepsilon_0 e^{-\alpha x^2}$ and using Eq. (11), we obtain the following function for index of refraction, $P(x)$ and $Q(x)$ as

$$n^2(x) = n_0^2 + \left(\frac{\alpha^2 - 1}{4}\right) x^2 + \alpha,$$

$$P(x) = e^{-\frac{1}{2}(\alpha + \frac{1}{2})} \alpha x^2 H_0 \left(x/\sqrt{2}\right),$$

$$Q(x) = e^{-\frac{1}{2}(\alpha + \frac{1}{2})} Q_0 \left(x/\sqrt{2}\right).$$

The similar results about this case can be seen in Fig. 3. Also, the description of obtained results is similar to the above-mentioned explanation for case a).

**Case c) $\varepsilon(x) = \varepsilon_0 (1 + \alpha x)^2$**

Using Eq. (11) and $\varepsilon(x) = \varepsilon_0 (1 + \alpha x)^2$, we obtain the following function for index of refraction, $P(x)$ and $Q(x)$ as

$$n^2(x) = n_0^2 + \frac{1}{4} \left[ \frac{\alpha^2 \beta (\beta + 2)}{(1 + \alpha x)^2} - x^2 \right],$$

$$P(x) = (1 + \alpha x)^{3/2} e^{-x^2/4} H_n \left(x/\sqrt{2}\right),$$

$$Q(x) = (1 + \alpha x)^{3/2} e^{-x^2/4} Q_n \left(x/\sqrt{2}\right).$$

The similar results about this case can be seen in Fig. 4. Also, the description of obtained results is similar to the above-mentioned explanation for case a).

**Case d) $\varepsilon(x) = \varepsilon_0 \cosh(\alpha x)$**

As a final example we consider $\varepsilon(x) = \varepsilon_0 \cosh(\alpha x)$ and using Eq. (11), we obtain the following function for index of refraction, $P(x)$ and $Q(x)$ as

$$n^2(x) = n_0^2 - \frac{1}{4} \left[ \frac{\alpha^2 - \cosh^2(\alpha x)}{\cosh^2(\alpha x)} + x^2 \right],$$

$$P(x) = \sqrt{\cosh(\alpha x)} e^{-x^2/4} H_n \left(x/\sqrt{2}\right),$$

$$Q(x) = \sqrt{\cosh(\alpha x)} e^{-x^2/4} Q_n \left(x/\sqrt{2}\right).$$

4. Conclusion

In this work, we present a new class of inhomogeneous media and its filtering properties were investigated. We show that the inhomogeneous medium length can be used for the bandwidth control effectively. The effect of cladding and substrate index of refractions on reflection and transmission coefficients and bandwidth of designed filter were analyzed. According to our obtained result, the length of inhomogeneous media is best way for bandwidth control.

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