Online small signal stability analysis of multi-machine systems based on synchronized phasor measurements

B. Mohammadi-Ivatloo *, M. Shiroei, M. Parniani

Center of Excellence in Power System Management and Control, Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran

A R T I C L E   I N F O

Article history:
Received 31 October 2010
Received in revised form 18 May 2011
Accepted 25 May 2011
Available online 22 June 2011

Keywords:
Power system stability
Phasor Measurement Unit (PMU)
Small signal stability
Dynamic model
Online monitoring
Wide area measurement

A B S T R A C T

This paper presents a novel approach for small signal stability assessment of a multi-machine system using only synchronized Phasor Measurement Units (PMUs) data. The proposed method does not need any information about the generators, network configuration or line impedances. By installing one PMU on each generator bus and using classical model for generator, all of the network and generators parameters needed for small signal stability analysis are estimated using the ambient data registered in the PMUs. Least square error estimation is used to obtain a reduced admittance matrix for the network in real-time. The estimated model is then used to evaluate the multi machine system dynamics in real-time. Simulation results show that the proposed method is applicable in practical systems and may be used to help the system operator to monitor small signal stability of the system.

© 2011 Elsevier B.V. All rights reserved.

1. Introduction

With rapid development of digital signal processing techniques and availability of Global Positioning System (GPS), PMU devices have been developed to give synchronized data from all over the power system. Sampled data of three-phase voltages and currents, which are time-synchronized, give the voltage and current magnitude and phase of each measured bus in the power system. After introduction and development of PMUs in late 1980s [1], a wide range of its applications in power systems is reported in the literature. Some of these applications are power system state estimation [2,3], wide area control and monitoring [4], fault location and detection [5,6], wide area protection [7], transient stability analysis and prediction [8], thermal monitoring of transmission lines [9], online steady state angle stability monitoring [10]. Other applications are also implemented in WECC and Brazilian power systems, which can be found in [11,12], respectively.

Small signal stability is an important aspect of power system dynamics. Conventional analysis methods use a linearized model of the system around certain operating point and perform stability analysis with state space modeling. These methods are based on complete physical model of the system and are suitable for offline studies. However, in practice power system experiences variations in both structure and operating point. Besides, the exact parameters of a power system may be unknown to us. Observing power system as a black box has been widely used in stability analyses. Most of the approaches make use of signal processing methods such as Prony analysis to analyze the output signals of the network. Most of these methods have been employed before the introduction of PMU devices and used local measurement devices to measure output signals of a single synchronous generator, e.g. its accelerating power. Prony analysis considers the studied system (here the power system) as an LTI system and assumes that each output signal of such a system is a summation of exponential terms. Then, these terms may be fitted to the measured signal by a least square error fitting to estimate residues and corresponding natural frequencies that exist in the output signal. In addition, known input signals may be injected in the system (e.g. voltage reference of AVR) to obtain transition function from that input to the measured output. The estimated transition function can be used for designing PSS [13].

Available papers in small signal stability monitoring and analysis using wide area PMU data can be divided into two groups. In the first group spectral analysis is performed on online wide area measured signals to obtain damping and oscillation frequencies. In [14] the eigenvalue and eigenvector of the interarea mode in Japan power system is estimated using Fourier spectrum analysis of signals measured by PMUs. In this paper deviation of phase angle between two places during steady-state conditions is measured by two PMUs connected to distribution system outlet. It is shown that
Fourier spectrum of the deviation has a clear peak corresponding to the interarea mode. In [15] Prony analysis accompanied with matrix pencil method and Hankel total least squares (HTLS) method is used to obtain electromechanical oscillation modes and mode shape of signals measured by PMUs. Hilbert spectral analysis is used in [16] to obtain dynamic characteristics of nonlinear oscillations recorded with PMUs. In [17] standard Hilbert–Huang transform (HHT) technique is refined to monitor the interarea oscillations more accurately. Spectral correlation analysis is used for estimating the mode-shape properties from PMU measurements in [18]. In this paper Periodogram averaging method is used to estimate the spectral properties and it is concluded that this method needs a minimum of 10–20 min of data to be implemented in an actual system. Prony analysis is widely used in monitoring low frequency oscillations in power system. In [19] Prony analysis is used to obtain low-frequency oscillation information from the signals measured by PMUs. A general assumption of Prony analysis is that the signal can be expressed by the summation of a series of complex exponential functions. With the reasoning that this assumption may not be correct for actual system signals, Wavelet based spectral analysis is used to monitor power system dynamics in real time in [20].

In the second group, measured signals are used to estimate specific transfer function parameters of the system, and then to obtain a model for stability analysis. In other words, a relation between injected signal and the system response is determined. In [21] low-order discrete-time model parameters for the power system are identified from background oscillations measured at two different locations. Then, eigenvalues of the identified discrete-time model are used to evaluate stability of the system. In [22] an autoregressive moving average exogenous (ARMAX) model is identified to represent measured data, and regularized robust recursive least squares (R3LS) method is used to estimate model parameters and electromechanical modes. An advantage of the spectral analysis is that the complete system model and other prior information about the system topology are not needed. However, these techniques are not able to determine the effects of different apparatus of the power system in the resulting dynamic characteristics. Therefore, they cannot be used directly for the purposes like controller design. On the other hand, in the transfer function identification methods, it is needed to inject a probing signal to the system and monitor the system output.

This paper adopts a combination of two approaches for small signal stability analysis. In the proposed method a reduced order model for power system is estimated from measured ambient signals without any input signal injection. In this method an equivalent classic model is estimated for each synchronous generator or a group of generators. Electrical parameter estimation is presented in [23] and is extended to mechanical parameter estimation in this work. All the electrical and mechanical parameters of the equivalent classic model are estimated using the measured PMU data. Also, the reduced admittance matrix of the network is estimated in real time without any knowledge of the network configuration or line impedances. Reduced admittance matrix is often used in stability analysis during transient state of the system. After estimating the required generators and network parameters, the system state matrix is calculated. Since the proposed method does not need any information about the network configuration and data, it will be applicable to any power system by installing a PMU on each generator terminal or generating plant substation. This condition may be relaxed as it will be relaxed later. The proposed method can be used for dynamic modeling of external systems as well. The estimated model has the capability of being used in power system controller design.

The remainder of the paper is organized as follows. In Section 2 equivalent generator model is extracted based on measured data. In Section 3 an algorithm for estimation of the system reduced admittance matrix is presented. Small signal stability analysis based on estimated models is presented in Section 4. Section 5 shows the case studies and simulation results. Finally, concluding remarks are presented in 6.

2. Estimating synchronous generator model parameters

In this section an equivalent two-order or classical model for synchronous generator will be estimated. Classical model assumes that the $d$–axis armature transient reactance and the internal emf representing the field flux does not change during transient period. Higher order models yield more accurate results at the cost of greater model complexity and required time to solve the differential equations. Classic model has two main advantages beside its simplicity. First, all the voltages and currents are phasors in the network reference frame, whereas in higher order models $d – q$ representation is needed [24]. The second important advantage is that the generator reactance can be treated in similar way as transmission line reactances and can be combined with network elements to form reduced admittance matrix. The latter advantage makes small signal stability analysis of multi-machine system much easier. It should be noted that while the estimated model is in the form of classical model, the estimated parameter values will differ from their physical values in a way to reflect the effect of higher order dynamics on electromechanical oscillations, as will be shown using simulation results. In other words, the presented model will benefit from advantages and simplicity of 2-order model while it considers the effect of neglected generator dynamics and its controls. Fig. 1 shows the classic model representation of the synchronous generator. For generator $i$, electrical parameters to be estimated are internal voltage ($E_i'$), transient reactance ($X_{dii}'$) and rotor phase angle ($\delta_i(t)$), and the mechanical parameters are damping coefficient ($K_{di}$) and inertia constant ($H_i$). It should be noted that the internal voltage magnitude is assumed to be constant while $\delta_i(t)$ can vary. For the sake of simplicity we define $\delta_i(t)$ as phase difference between bus i terminal voltage and internal voltage phase angles, i.e.

$$\delta_i(t) = \delta_{i0} + \theta_i(t)$$  \hspace{1cm} (1)

where $\theta_i(t)$ is a bus terminal voltage measured by PMU and $\delta_i(t)$ is a generator internal voltage phase angle which should be estimated. The generator output active and reactive powers can be written as (2) and (3), respectively.

$$P_i = \frac{E_i' V_i' \sin(\delta_i)}{X_{dii}'} \hspace{1cm}$$ \hspace{1cm} (2)

$$Q_i = \frac{V_i(E_i' \cos(\delta_i) - V_i)}{X_{dii}'} \hspace{1cm}$$ \hspace{1cm} (3)

Using m sets of measurement data provided by a PMU device placed in the terminal of generator $i$ results in 2m nonlinear equations. Eqs. (2) and (3) may be rewritten as follows for $m$ sets of measurement data:

$$p_k X_{dii}' E_i' \sin(\delta_i) = 0 \hspace{1cm} k = 1, 2, \ldots, m$$  \hspace{1cm} (4)

$$Q_i X_{dii}' - V_i^k E_i' \cos(\delta_i) + (V_i^k)^2 = 0 \hspace{1cm} k = 1, 2, \ldots, m$$  \hspace{1cm} (5)

In (4) and (5) values of $Q_i^k$, $V_i^k$ and $p_k$ are known and values of $E_i'$, $X_{dii}'$ and $\delta_i^k$ are unknown. Then there will be $m + 2$ unknowns for
2m equations. Determining unknown parameters can be stated as optimization problem (6) for m greater than 2.

\[
\min_{E_i, X_{di}, \delta_i(k \Delta t)} \begin{bmatrix}
    P_i(k \Delta t)X_{di} - E_iV_i(k \Delta t)\sin(\delta_i(k \Delta t)) \\
    Q_i(k \Delta t)X_{di} - V_i(k \Delta t)E_i\cos(\delta_i(k \Delta t)) + V_i(k \Delta t)^2 \\
    \cdots \\
    P_i(k \Delta t)X_{di} - E_iV_i(k \Delta t)\sin(\delta_i(k \Delta t)) \\
    Q_i(k \Delta t)X_{di} - V_i(k \Delta t)E_i\cos(\delta_i(k \Delta t)) + V_i(k \Delta t)^2
\end{bmatrix}
\]

This optimization problem should be solved for all generators separately. The nonlinear least square method can be used to estimate unknown parameter values. The vector of unknown parameters \( \mathbf{x} \) for each generator is defined as follows:

\[
\mathbf{x}_i = \begin{bmatrix}
    E_i \\
    X_{di} \\
    \delta_i(k \Delta t) \\
    \cdots \\
    \delta_i(m \Delta t)
\end{bmatrix}^T
\]

Then, \( \mathbf{x}_i \) is calculated iteratively as

\[
\mathbf{x}_{i+1} = \mathbf{x}_i + \Delta \mathbf{x}_i
\]

where \( \Delta \mathbf{x}_i \) is equal to

\[
\Delta \mathbf{x}_i = -[\mathbf{J}_i]^{-1} \mathbf{F}_i(k \Delta t)^{\text{T}} - \mathbf{P}_i(k \Delta t)
\]

In (9), \( \mathbf{F}_i \) is an \((m+2) \times 1\) vector and the Jacobian matrix \( \mathbf{J}_i \) is a \((2m) \times (m+2)\) matrix defined as follows:

\[
\mathbf{F}_i(k \Delta t) = \frac{\partial L}{\partial \mathbf{x}_i} = \begin{bmatrix}
    E_iV_i \sin(\delta_i) - P_i(k \Delta t) \\
    E_iV_i \cos(\delta_i) - Q_i(k \Delta t) - V_i(k \Delta t)^2
\end{bmatrix}
\]

This equation is transformed to discrete form using Tustin or Bilinear method. In this method differential operator \((d/dt)\) is substituted with \((2/\Delta T)[Z - 1/Z + 1]\), where \( Z \) is operator of z-transform and \( \Delta T \) is sampling period. The discrete form of swing equation is:

\[
\frac{2H_i \omega_0}{\Delta T}[\delta_i(k + 2) - 2\delta_i(k + 1) + \delta_i(k)] + \frac{K_{di} \omega_0}{\Delta T}\delta_i(k)
\]

where \( P_i \) is acceleration power and equals to the difference of input mechanical power and output electrical power \((P_{mi} - P_{ei})\). \( \delta_i(k) \) in (13) are already estimated from (8). Then, \( K_{di} \) and \( H_i \) can be calculated using linear least square optimization of (13).

3. Network parameters estimation

In stability assessment of multi-machine systems, the following simplifying assumptions are often made:

1. Each synchronous machine is represented by a constant internal voltage behind the transient reactance.
2. The governor’s actions may be neglected. It means that input mechanical power of turbine generator remains constant during and after the disturbance.
3. During the transients, loads are modeled as constant admittances to ground.

By these assumptions the system can be modeled as shown in Fig. 2. To identify the reduced admittance matrix of the system, the admittance matrix corresponding to transmission network and loads is first determined using the measurement data at generator terminals. Since no current is injected to load buses, the only current injections to the network are at the generators terminals. As it shown in Fig. 2 buses from 1 to \( N_g \) are generator buses and buses from \( N_g + 1 \) to \( N \) are load buses. The no load voltage equation of the system is then as follows:

\[
\begin{bmatrix}
    I_g \\
    0
\end{bmatrix} = \begin{bmatrix}
    Y_1 & Y_2 & \cdots & Y_{N_g} \\
    Y_{N_g + 1} & \cdots & Y_N
\end{bmatrix} \begin{bmatrix}
    V_g \\
    \cdots \\
    V_N
\end{bmatrix}
\]

where \( I_g = (I_1 \ldots I_{N_g})^T \), \( V_g = (V_1 \ldots V_{N_g})^T \) and \( V_i = (V_{N_g + 1} \ldots V_N)^T \).
It should be noted that similar to synchronous generator modeling, it is possible to estimate dynamic load models using PMU data with proper formulation. The Kron reduction method can be applied to obtain reduced admittance matrix using (14).

\[ I_g = Y_{\text{red}} V_g \]  \hspace{1cm} (15)

where \( Y_{\text{red}} = Y_1 - Y_2 Y_3^{-1} Y_3 \). Our objective is to estimate the reduced admittance matrix based on measurements without having any information about network topology and parameters. Eq. (15) can be rewritten as follows for \( m \) sets of voltage and current phasor measurements:

\[
\begin{bmatrix}
I_{g1}^T & I_{g2}^T & \cdots & I_{gm}^T
\end{bmatrix} = Y_{\text{red}} \begin{bmatrix}
V_{g1}^T & V_{g2}^T & \cdots & V_{gm}^T
\end{bmatrix}
\]  \hspace{1cm} (16)

The elements of reduced admittance matrix can be readily obtained using least square estimation method as follows:

\[
Y_{\text{red}} = \begin{bmatrix}
I_{g1} & I_{g2} & \cdots & I_{gm}
\end{bmatrix} \begin{bmatrix}
V_{g1} & V_{g2} & \cdots & V_{gm}
\end{bmatrix}^{-1}
\]  \hspace{1cm} (17)

After this stage, the previously estimated generators reactances can be added to diagonal elements of \( Y_{\text{red}} \) to obtain the reduced admittance matrix of the overall system \( Y_{\text{red}} \). Alternatively, it is possible to use estimated \( E_i \angle \delta_i \) instead of \( V_i \angle \theta_i \) in (17) to obtain \( Y_{\text{red}} \) directly.

4. Small signal stability analysis of multi-machine power system

Estimation of the required parameters for small signal stability analysis of a multi machine system was presented in the previous section. In this section, formulation of the small signal stability problem using the obtained parameters will be presented. Small signal stability analysis deals with perturbations around the operating point and whether these perturbations result in instability of the system or not. Every linearized dynamic system may be represented by state equations \( \dot{X} = AX \) in which \( X \) is the vector of state variables. For the purpose of stability analysis we need to obtain the state matrix \( A \) and its eigenvalues. The linearized dynamic equations of each synchronous generator can be written as follows:

\[
\frac{d(\Delta \delta_i)}{dt} = \Delta \omega_i
\]  \hspace{1cm} (18)

\[
\frac{2H_i}{\omega_s} \frac{d(\Delta \omega_i)}{dt} = -\Delta P_{ei} - K_{di} \Delta \omega_i
\]  \hspace{1cm} (19)

where \( \Delta P_{ei} = \sum_{j=1}^{N_g} (\partial P_{ei}/\partial \delta_j) \Delta \delta_j \) and

\[
\frac{\partial P_{ei}}{\partial \delta_j} = \begin{cases} E_i E'_{ij} \sin(\delta_i - \delta_j - \phi_{ij}) & j \neq i \\ - \sum_{j=1}^{N_g} E_i E'_{ij} \sin(\delta_i - \delta_j - \phi_{ij}) & j = i \end{cases}
\]  \hspace{1cm} (20)

where \( E'_{ij} \angle \phi_{ij} \) is the element \( ij \) of \( Y_{\text{red}} \). The multi-machine system dynamic equations can be written in a state space format as follows:

\[
\frac{d}{dt} \begin{bmatrix}
\Delta \delta_1 \\
\Delta \delta_2 \\
\vdots \\
\Delta \delta_{N_G}
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
0 & A_{\omega} \\
\vdots & \vdots \\
0 & A_{k}
\end{bmatrix} \begin{bmatrix}
\Delta \omega_1 \\
\Delta \omega_2 \\
\vdots \\
\Delta \omega_{N_G}
\end{bmatrix}
\]  \hspace{1cm} (21)

where \( I \) is a unity matrix of order \( N_G \), \( A_{\omega} \) and \( A_k \) submatrices can be calculated using (22) and (23), respectively.

\[
A_{\omega} = \begin{bmatrix}
-\frac{\omega_s}{2H_1} & \frac{\partial P_{e1}}{\partial \delta_1} & \cdots & -\frac{\omega_s}{2H_1} & \frac{\partial P_{e1}}{\partial \delta_{N_G}} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
-\frac{\omega_s}{2H_N} & 0 & \cdots & -\frac{\omega_s}{2H_N} & K_{d1}
\end{bmatrix}
\]  \hspace{1cm} (22)

\[
A_k = \begin{bmatrix}
0 & 0 & \cdots & 0 & -\frac{\omega_s}{2H_N} K_{dN_G}
\end{bmatrix}
\]  \hspace{1cm} (23)

All the required parameters in matrix \( A \) are estimated in previous sections. It is then straightforward to compute eigenvalues of \( A \) and evaluate small signal stability of the multi-machine system. More details can be found in [25].
5. Case studies and simulation results

In order to evaluate the effectiveness of the proposed method three different test systems are studied. At each case, the system response to a disturbance is simulated with the given machine and network parameters. Except for the first test case, complete models of generators are used in the simulations, to resemble the actual system behavior. Samples of the simulated response are used instead of PMU measurement data for purpose of model identification as described earlier. Dynamic characteristics of the identified model are then compared with those of original system. It should be noted that in the case of using real PMU data, proper preprocessing and filtering is needed. Extraneous data outliers and high order components have to be filtered out before implementing the proposed method.

5.1. Single machine infinite bus (SMIB) test system

Fig. 3 shows the SMIB test system. Generator parameters adopted from [26] are presented in Table 1. Generator reactances and its armature resistance are in per unit. Two models for synchronous generator are used. In the first case, to examine the accuracy of estimation method, the machine is represented with classical model. In the second case, detailed generator model considering d and q axes damper windings is used. To simulate a disturbance on the system, a three phase fault with duration of 0.05 s is applied at t = 2 s on the sending end of one line connecting the machine to infinite bus. It is expected that the parameters estimated using the simulation result samples be identical to the original parameters in the first case. To simulate the PMU data flow, the sampling rate is selected to be 30 samples/s. In practice the sampling rate of PMU varies from 12 samples/s to 60 samples/s. By changing the sampling rate it is observed that the estimation performance has negligible sensitivity to the change of sampling rate in this range. Therefore, in the remainder of this paper, the sampling interval is selected to be 30 samples/s.

Table 2 shows the parameter estimation results for two cases. As it can be observed, the estimation results for case I (classic generator model) is approximately identical to original values. This is expected because the original and the identified models are the same. In contrast, the results for case II (detailed generator model) are quite different because the original model and identified model are different. This difference is in a way that allows the low order estimated model to represent the original system behavior more accurately. This will be shown with eigenvalue analysis and time domain simulations.

Table 3 shows the damping (α), frequency (f) and damping ratio (ξ) of electromechanical modes obtained using original and estimated models. As expected the estimated and original model results for case I are very close. It can be observed that for case II the results of estimated model are much closer to the detailed model than original classic model results. This confirms that the difference between estimated and original classic model parameters are in a way to represent the effects of higher order dynamics on electromechanical oscillations. Fig. 4 shows the output power of the generator simulated with the original classic and detailed models as well as the estimated model. The calculation time with Matlab running on an ordinary PC was 400 ms. It is observed that the response of estimated model is very close to detailed model response as compared to the original classic model.

5.2. 4-Machine test case

Fig. 5 shows a 2-area 11-bus test system used for multi-machine system stability analysis [27]. Four PMUs are placed on generator terminal buses (1, 2, 11 and 12). It must be noted here that supposing PMU placement on generator terminals is not a necessary condition of the proposed method. If PMUs are placed on non-generating buses (as in many actual installation), with the proposed method we calculate equivalent generator models viewed from these buses.

To simulate a disturbance on the system, a three phase fault with duration of 0.05 s is applied at t = 2 s on the sending end of one of the lines connecting bus 3 to bus 101. Six-order synchronous generator model with one damper winding on d-axis and two on q-axis was used to represent generators dynamics. All the generators were supposed to be equipped with excitation system, governor and power system stabilizer (PSS). The simulated system data can be found in [28]. Table 4 shows the results of generator parameter estimation for this system. It can be seen that the estimated parameters are quite different from original classic model values in some cases. To examine suitability of the estimated model, small signal stability analysis is done both for the system with detailed model of generators and their controls, and the identified model. The analy-
sis is also performed on the original system with classical generator models to compare the results. The detailed system model order is 60. The estimated model is of order 8. Fig. 6 compares eigenvalues of electromechanical modes for detailed model, original classic model and the estimated model. The calculation time for this case is 900 ms. While the classic model does not depict the system instability, the estimated model captures this characteristic. The estimated parameters of the generators are presented in Table 5. Fig. 7 shows the obtained modes using the three models. It is observed that unlike classic model, the estimated model succeeded to detect the instability.

5.2.2. Effect of noise and ambient data
An important concern in the estimation process is the performance of the presented method in the presence of noise and ambient data. Simulations are carried out on the 4-machine test

### Table 4
Generator parameter estimation results for 4-machine test system.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$X'_d$ (PU)</th>
<th>$H$</th>
<th>$K_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original value</td>
<td>0.25</td>
<td>6.5</td>
<td>10</td>
</tr>
<tr>
<td>Estimated value (generator 1)</td>
<td>0.1336</td>
<td>8.1965</td>
<td>13.5871</td>
</tr>
<tr>
<td>Estimated value (generator 2)</td>
<td>0.0973</td>
<td>6.2066</td>
<td>13.7512</td>
</tr>
<tr>
<td>Estimated value (generator 3)</td>
<td>0.4948</td>
<td>6.1331</td>
<td>12.4302</td>
</tr>
<tr>
<td>Estimated value (generator 4)</td>
<td>0.4515</td>
<td>6.5369</td>
<td>16.4101</td>
</tr>
</tbody>
</table>

5.2.1. Estimating unstable modes
In this section the 4-machine test case [28] without PSS is studied to examine the capability of the proposed method in estimating unstable modes. Applied disturbance is similar to previous case. Small signal stability analysis of the detailed model, which is of order 44, shows that the system is unstable. The total calculation time for this case is 880 ms. While the classic model does not depict the system instability, the estimated model captures this characteristic. The estimated parameters of the generators are presented in Table 5. Fig. 7 shows the obtained modes using the three models. It is observed that unlike classic model, the estimated model succeeded to detect the instability.
system to investigate the effect of these concerns. To simulate the
effect of measurement noise, random 0.01% multiplicative errors
are added to the amplitudes and phase angles of the voltage and
current phasors. This assumption corresponds to the noise levels
specified by manufacturers of PMUs [29]. For generation of ambient
data, random load changes are produced and fed into the loads at
buses 4 and 14 with the maximum level of 2%. Before identification
of the system dynamic model, the simulation data are low-pass fil-
tered by a fourth order Chebyshev filter with cut off frequency of

5 Hz. Fig. 8 shows the bus 4 noisy and ambient voltage before and
after filtering. Table 6 shows the results of generator parameter esti-
mation for this case. Fig. 9 compares the electromechanical modes
with the detailed, classic and the estimated models. Comparison
between the estimation results obtained from ambient data and the
results obtained from a major system perturbation shows that the
estimation accuracy is degraded when the perturbations and
oscillations are very small. In the case of too small ambient data,
the algorithm may diverge. This problem is also reported in the previous works [23]. To preserve the model accuracy and to avoid divergence problem, a trigger algorithm can be employed to activate the estimation process upon sufficiently large perturbations.

5.3. 16-Machine test system

The third studied system is a 16-machine 68-bus test system. The system data are adopted from [30]. All the generators are supposed to be equipped with excitation system and governor. Generators 1–12 are equipped with power system stabilizer (PSS). Detailed system model is used to generate the sampled simulation data. The order of detailed model is 196. The order of the estimated model is 32. To simulate a disturbance on the system, a three phase fault is applied at bus 1, on line 1–2 at t = 1 s for duration of 0.05 s. The results of generator parameter estimation for this system are presented in Table 7. In order to compare the results of the estimated model, electromechanical modes of the system are calculated for the estimated model and compared with those of the detailed model, as well as the classic model with actual parameters, in Fig. 10. The calculation time is 1.5 s. It is observed that the estimated model yields much closer results to the measured high-order system than the classic model.

5.3.1. Estimating unstable modes

In order to study the performance of the proposed method on estimating unstable modes of 16-machine test system, PSS of machines 9–12 are removed. The other parameters and applied disturbance are same to previous section. The order of the detailed model in this case is 184 and small signal stability analysis shows that the system has two unstable modes. The estimated parameters of the generators are presented in Table 7 and obtained modes using the three models are compared in Fig. 11. The calculation time is 1.5 s. Using classic model does not depict the system instability but estimated model is able to detect system instability.

![Fig. 8. Bus 4 voltage before and after filtering.](image1)

![Fig. 9. Comparison of electromechanical modes for 4-machine test system with noisy ambient data.](image2)
Table 7
Generator parameter estimation results for 16-machine test system.

<table>
<thead>
<tr>
<th>Generator</th>
<th>Original classic model values</th>
<th>Estimated values</th>
<th>Estimated values (unstable case)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$X_d'$</td>
<td>$K_d$</td>
<td>$H$</td>
</tr>
<tr>
<td>1</td>
<td>0.248</td>
<td>3.4</td>
<td>3.4</td>
</tr>
<tr>
<td>2</td>
<td>0.42529</td>
<td>4.9494</td>
<td>4.9494</td>
</tr>
<tr>
<td>3</td>
<td>0.38309</td>
<td>4.9623</td>
<td>4.9623</td>
</tr>
<tr>
<td>4</td>
<td>0.29595</td>
<td>4.1629</td>
<td>4.1629</td>
</tr>
<tr>
<td>5</td>
<td>0.36</td>
<td>4.7667</td>
<td>4.7667</td>
</tr>
<tr>
<td>6</td>
<td>0.35433</td>
<td>4.9107</td>
<td>4.9107</td>
</tr>
<tr>
<td>7</td>
<td>0.29898</td>
<td>4.3267</td>
<td>4.3267</td>
</tr>
<tr>
<td>8</td>
<td>0.3579</td>
<td>3.915</td>
<td>3.915</td>
</tr>
<tr>
<td>9</td>
<td>0.48718</td>
<td>4.0365</td>
<td>4.0365</td>
</tr>
<tr>
<td>10</td>
<td>0.48675</td>
<td>2.9106</td>
<td>2.9106</td>
</tr>
<tr>
<td>11</td>
<td>0.25312</td>
<td>2.0053</td>
<td>2.0053</td>
</tr>
<tr>
<td>12</td>
<td>0.5248</td>
<td>5.1791</td>
<td>5.1791</td>
</tr>
<tr>
<td>13</td>
<td>0.3446</td>
<td>8</td>
<td>4.0782</td>
</tr>
<tr>
<td>14</td>
<td>0.285</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>15</td>
<td>0.285</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>16</td>
<td>0.35899</td>
<td>8.9</td>
<td>4.45</td>
</tr>
</tbody>
</table>

Fig. 10. Comparison of electromechanical modes for 16-machine test system.

Fig. 11. Comparison of electromechanical modes for 16-machine test system (unstable case).
6. Conclusion

A novel method for online assessment of small signal stability of large systems is presented. With this method, there is no need to have any information about network topology and turbine-generator parameters. The proposed method estimates the dynamic behavior of the system in real-time, allowing it to resist against changes in the network configuration and operating condition. The proposed method identifies a 2-order equivalent generator model viewed from each PMU location. The system reduced admittance matrix is also estimated using PMU data considering the system loads as constant impedances. Simulation results on SMIB and multi-machine examples showed that the low order estimated model represents the actual system dynamics with good approximation, and reflects the effects of neglected dynamics on electromechanical oscillations. It should be noted that the location of the disturbance has no considerable effect on the estimated results. The proposed method can be used by system operators to monitor small signal stability margin of the network.

References